**3-Rotation**

Rotation about origin through an angle is a transformation defined as:

Where

* If the direction of is not defined, then it is understood to be in anticlockwise direction.
* If is in clockwise direction, then replace by in the above definition as:

**Example 1:** Sketch the image of given rectangle with vertices A(0,0), B(3,0), C(3,2), D(0,2) under the rotation of (anticlockwise).

**Solution:** As the transformation of rotation is

Where

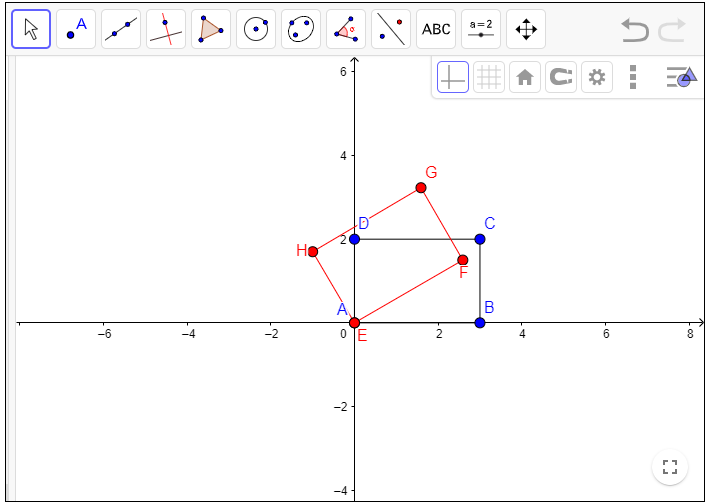
As , so

For point A:

For point B:

For point C:

For point D:



**Work to do:**

**Q1.** Sketch the image of given parallelogram with vertices A(0,1), B(3,0), C(5,-2),

D(2,-1) under the rotation of (anticlockwise) .

**Q2.** Sketch the image of given triangle with vertices A(2,4), B(2,2), C(4,2) under the

rotation of 9 (clockwise) .

**Example 2.** Lety = 2x+5 be a line. Find the equation of line after rotating it through

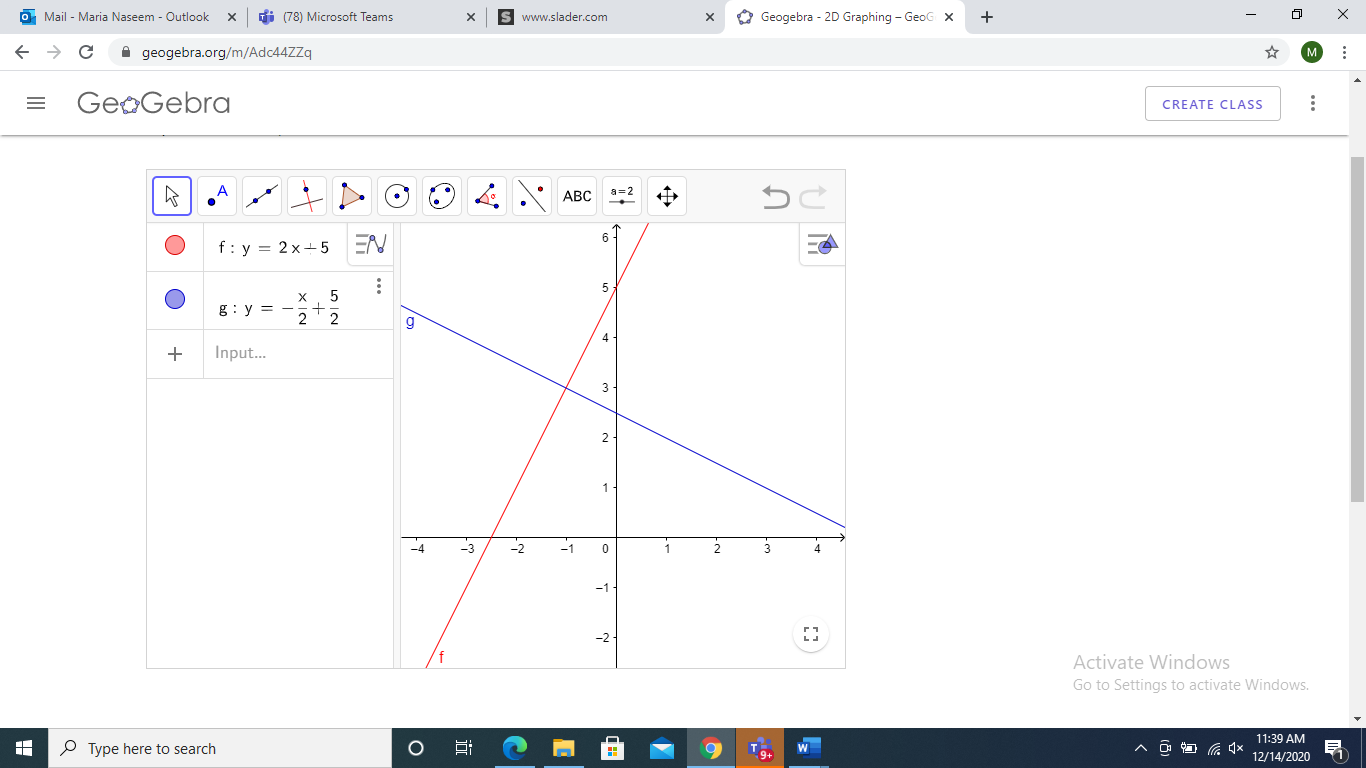
an angle of clockwise direction about origin.

**Solution:** The matrix of rotation in clockwise direction is

As , so

Put value of x and y in original equation of line and obtain

This is the rotated line with angle in clockwise direction.



**Work to do:**

**Q3.** Let be a line. Find the equation of line after rotating it through an angle of clockwise direction about origin.

**Example 3.** Let be a circle. Find the equation of circle after rotating it through an angle of in anticlockwise direction about origin.

**Solution:** The matrix of rotation in anticlockwise direction is

As , so

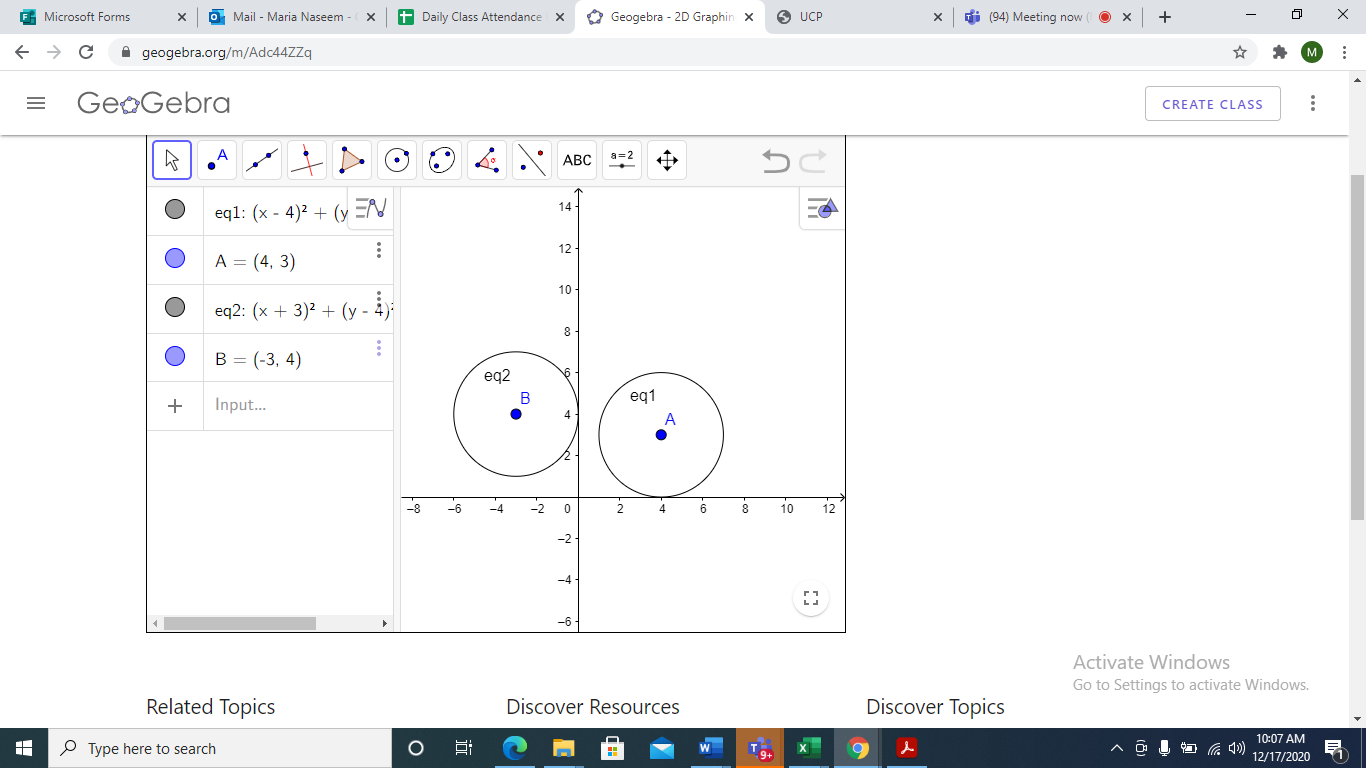
Therefore, for

Or

Putting these values of x and y in original equation of circle

We get

This is the equation of rotated circle with angle in anticlockwise direction.



**Work to do:**

**Q4.** Let be a circle. Find the equation of circle after rotating it through an angle of in clockwise direction about origin.

**Example 4:** Let be an ellipse. Find the equation of ellipse after rotating it through an angle of in anticlockwise direction about origin.

**Solution:** The transformation of rotation in anticlockwise direction is

As

Hence, or

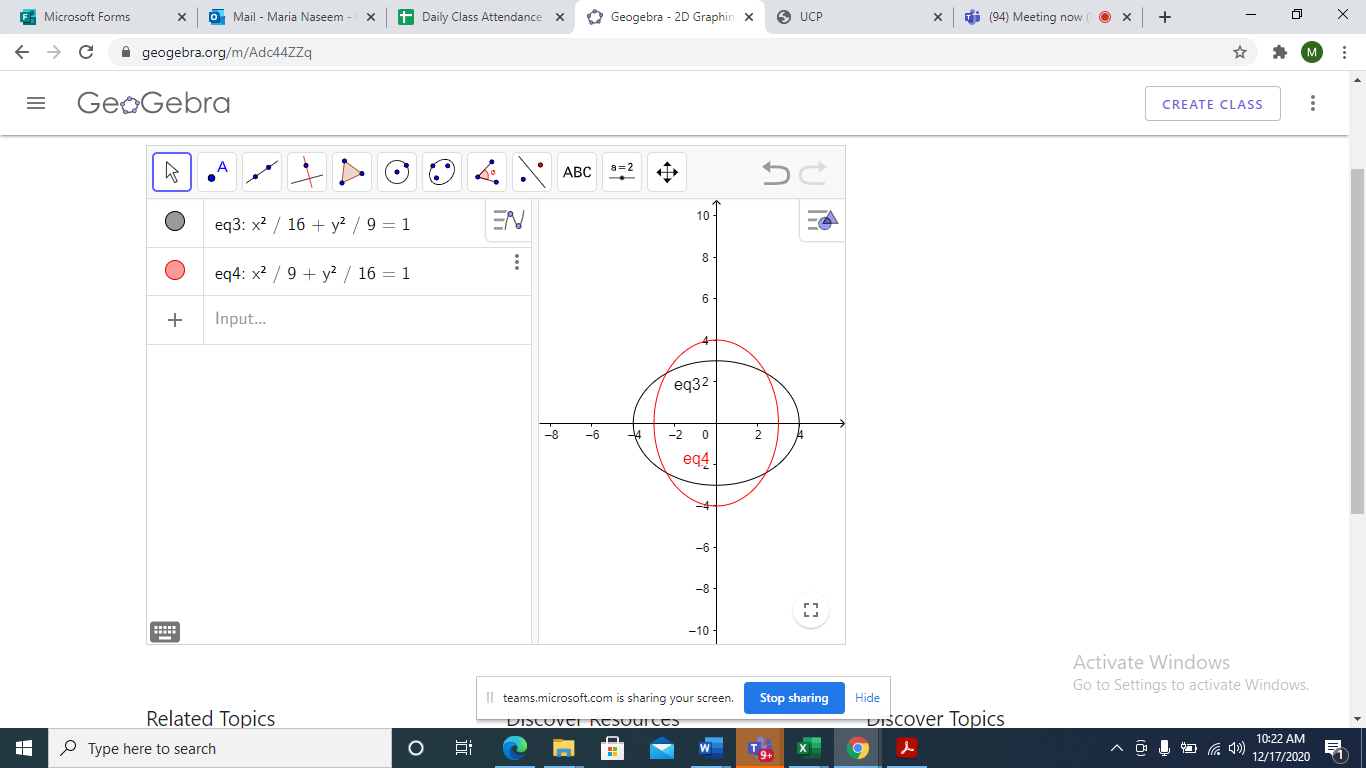
Put these values of x and y in original equation of ellipse, we get the rotated ellipse with angle in anticlockwise direction as

For plotting we can neglect the dash from our rotated equation of ellipse.

**Original Ellipse Rotated Ellipse**

**Major axis is along x-axis** **Major axis is along- yaxis**

,



**Q 5.** Let be an ellipse. Find the equation of ellipse after rotating it through an angle of in anticlockwise direction about origin.

**Affine Transformations**

An affine transformation is a transformation , defined by

For some , where is invertible matrix i.e.

**Types of Affine Transformation:**

1. **Scaling**

A transformation , defined by:

Where

Is said to be a scaling by the factor k.

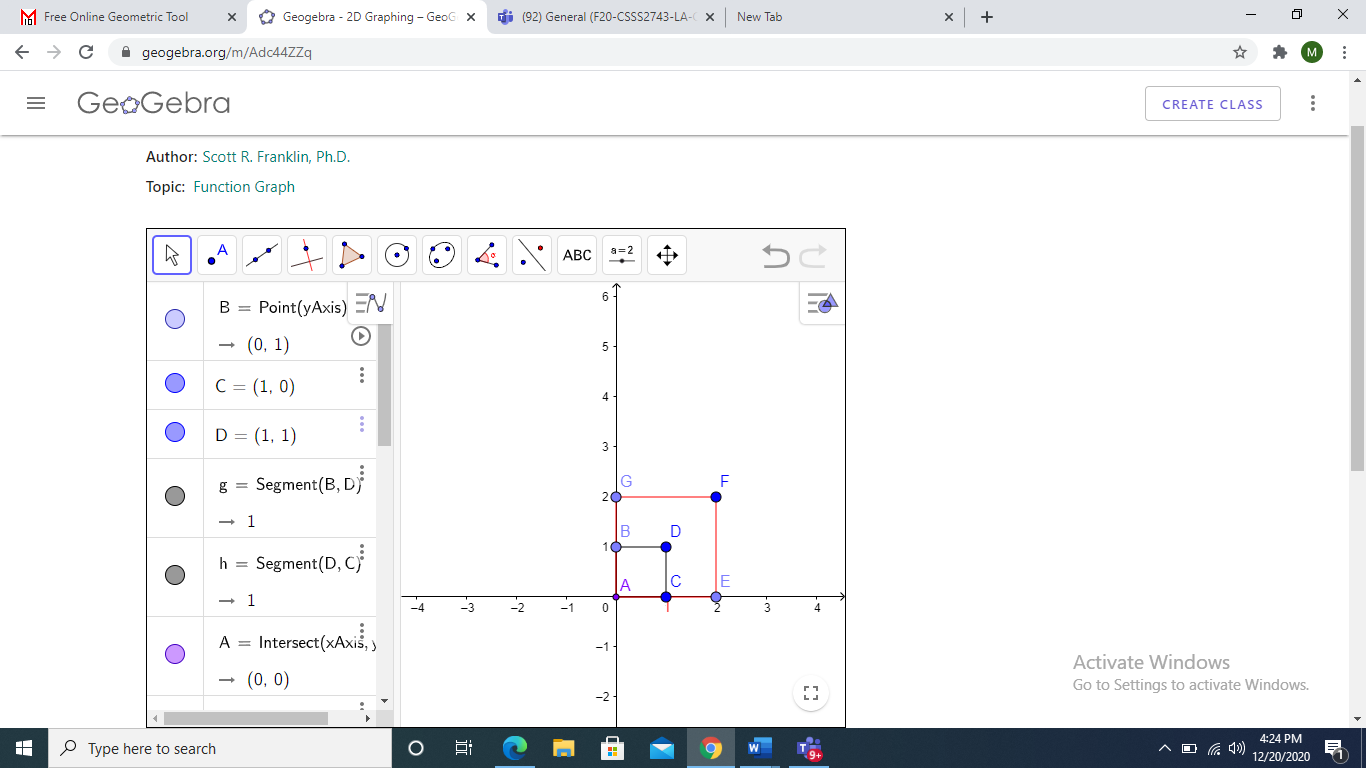
**Note:**

1. If k>1, then its dilation/enlargement.
2. If then its contraction.

**Example 1: (Scaling of Square)**

A square with vertices . Scale this square by factor 2.

**Solution:**

As

**Example 2: (Scaling of Line)**

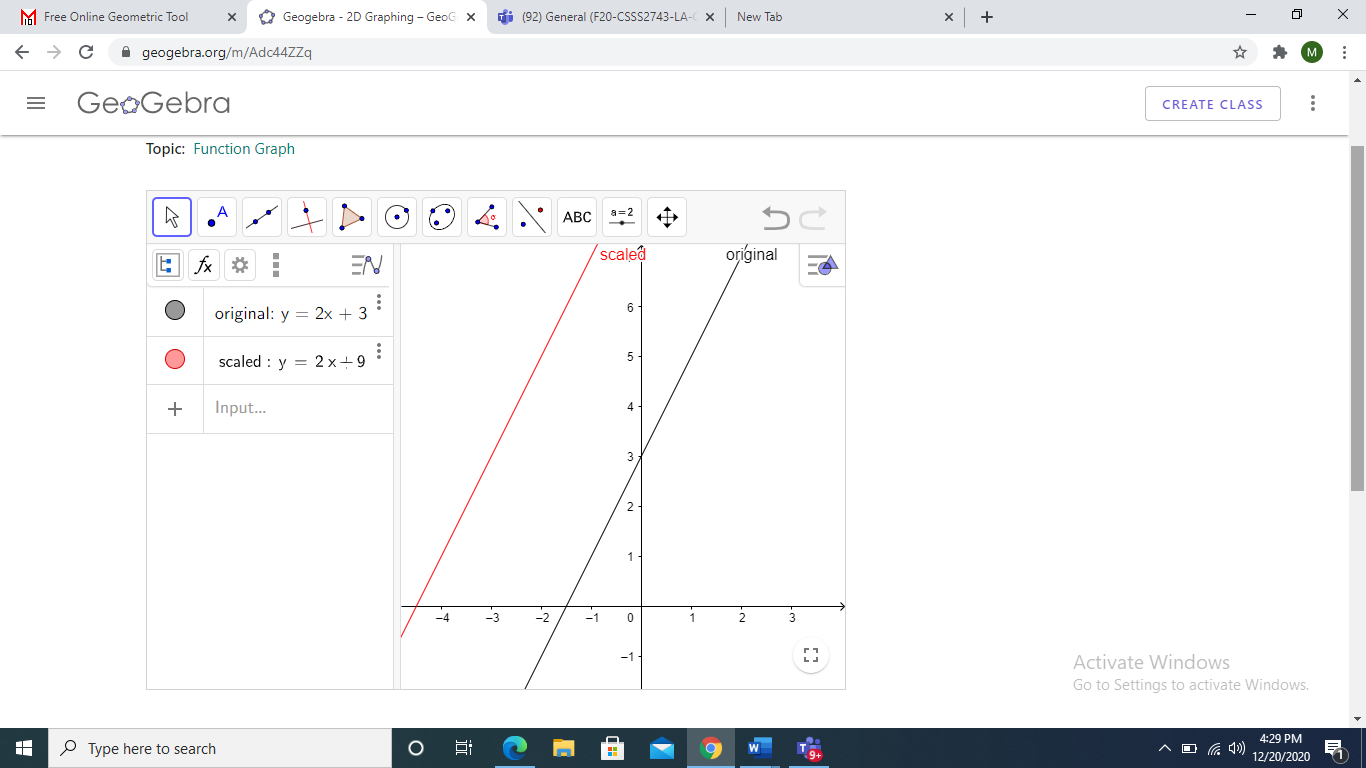
Let be a line. Determine the image of this line under the scaling by factor 3.

**Solution:**

As

Put value of and in original equation of line:

And get the equation of scaled line with factor 3.



**Example 3: (Scaling of Circle)**

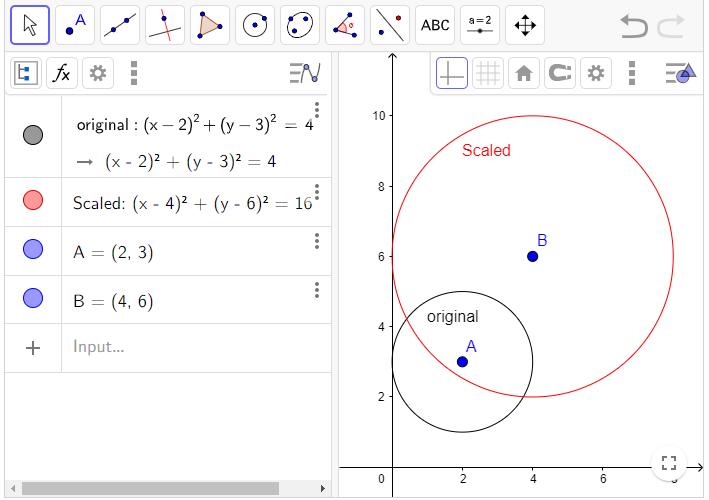
Determine the image of the circle

under the scaling by factor 2.

**Solution:** As

Put value of and in original equation of circle:, implies

This is the equation of circle by factor 2.



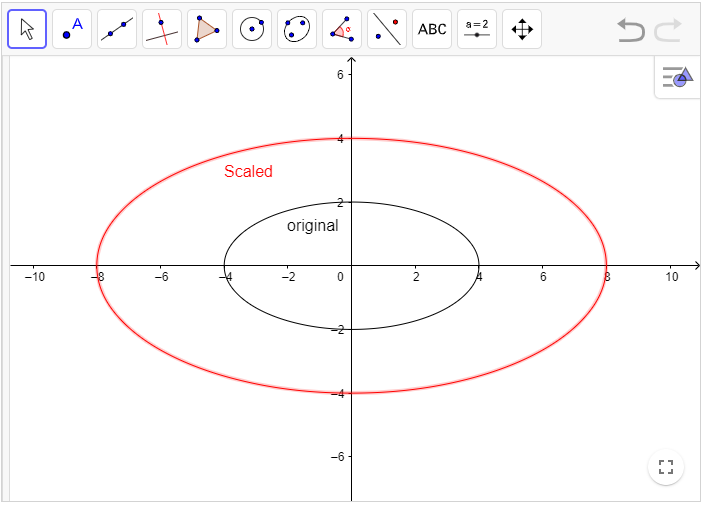
**Example 5:** (Scaling of an Ellipse)

Determine the image of an ellipse

under the transformation of scaling by factor 2.

**Solution:** As

Put value of and in original equation of ellipse and get an ellipse scaled by factor 2.



**Practice Problems**

Q1. Find the Image of a triangle with vertices A= (2, 2), B = (4, 3), C = (5,5), when these are scaled by factor 2.

Q2. Find the Image of line under the transformation of scaling by factor 3.

Q3. Find the image of an ellipse under the transformation of scaling by factor 2.